International Journal of Theoretical Physics, Vol. 25, No. 3, 1986

Quantum Gravity and the Statistical Interpretation of Quantum Mechanics

Lee Smolin[1]

Received September 9, 1985

Three independent arguments are given for the conclusion that the distinction between quantum fluctuations and real statistical fluctuations in the state of a system will not be maintained in a theory that gives a correct description of phenomena in which quantum and gravitational effects are both important. As this distinction is absolute in terms of the orthodox interpretation of the quantum state something in either the interpretation of the quantum state or the interpretation of the thermodynamic state will have to be altered to construct a theory which describes both quantum and gravitational phenomena. I propose that we pursue the simplest possibility, which is to adopt the statistical interpretation of the wave function in which quantum fluctuations are understood to be ordinary statistical fluctuations in an ensemble of individual physical systems.

1. INTRODUCTION

Among the interpretations of quantum mechanics which have been proposed in the last 60 years are some which might be termed dogmatic, and some which might be termed provisional. In the first category are those interpretations the assertion of which requires that quantum mechanics be the final physical theory. The chief reason for an interpretation to be in this category is that it includes the assertion that a state vector in Hilbert space gives the complete description of an individual physical system. Examples of interpretations which fall in this category are Bohr's interpretation (Bohr, 1934; von Neumann, 1955) and the many-worlds interpretation (Everett, 1957; DeWitt and Graham, 1973).

Provisional interpretations, on the other hand, are those that allow us to compare calculations done in the quantum theory with experiment, without requiring any assertion as to the ultimate validity of the Hilbert space formalism as a complete description of physics. As every interpretation includes some prescription for connecting calculation with experiment, and

---

[1] Department of Physics, Yale University, New Haven, Connecticut 06511.

0020-7748/86/0300-0215$05.00/0

as this prescription cannot depend on metaphysical assertions, every dogmatic interpretation has a weak form which is provisional. For example, in the case of Bohr's interpretation the weak form is the set of rules, deriving from the canonical commutation relations and the uncertainty principle, which govern what question can and cannot be asked in quantum mechanics. (Indeed, it is this, and not the much more radical epistemology of Bohr, which is usually taught in textbooks under the name of the standard or Copenhagen interpretation.) Weak forms of the many-worlds interpretation which are not dogmatic have been proposed by Geroch (1984) and the author (Smolin, 1984a).

In addition there are interpretations which to begin with are provisional. Among these is the statistical interpretation, which asserts that the state function describes a statistical ensemble of similarly prepared systems (Ballentine, 1970). As the statistical predictions of quantum mechanics can only be checked against measurements made on such ensembles, this assertion cannot be in contradiction with any experiment which is taken as a confirmation of quantum theory. At the same time, this interpretation will remain viable, whatever the future holds for quantum mechanics as a fundamental theory.

What has all this got to do with quantum gravity? One might suppose that the question of the interpretation of quantum mechanics is completely irrelevant to the effort to unify quantum mechanics with general relativity. Certainly, an increasing number of phenomena have been incorporated into the quantum theory without there being the slightest effect on the problem of the interpretation of quantum mechanics. However, there are two features of gravitational theory which distinguish it from the theories of other interactions, which make it possible that the effort to quantize gravity may become entangled with the muddle concerning the interpretation of quantum mechanics.

First, both general relativity and quantum mechanics are what Einstein called theories of principle (Einstein, 1949). By a theory of principle, he meant a theory which governs, not just some particular phenomena, but the framework into which any theory of any given phenomena must fit, if it is to be sensible. Theories of principle have the feature that they cannot be true if they apply to certain phenomena and not to others. This is because it can be shown that if there are some particles or fields to which they do not apply which interact with some to which they do apply, then one can construct contradictions of the theory's basic principles. This is known to be the case for both general relativity and quantum mechanics.

When we have a confrontation between two theories of principle, as we do in the attempt to quantize gravity, one of several things may happen. It may turn out that one or another of the two theories is found not to be universally applicable, so that it must suffer modification before it can be inserted in the universal framework provided by the other. It may turn out that neither is universally applicable, so that the unified theory that joins them requires important modification in the principles of each. Or, finally, it may turn out that the principles of both theory are not in conflict, so that they can be joined essentially without modification.

It is clear that if quantum mechanics is found not to have universal validity, the dogmatic interpretations will fail. However, even if general relativity is brought into quantum theory in a way that does not require modification of the axioms of quantum mechanics, the result may still be that some of the dogmatic interpretations of quantum mechanics are no longer applicable to the resulting theory.

This is because of the peculiar situation that general relativity includes in its domain cosmology. Any successful unification of general relativity with quantum mechanics should give us quantum descriptions of those space-times which are asserted to be models for the entire universe. However, this comes into conflict with the separation, required by some interpretations of quantum mechanics, between the system which is being studied, and the observer and measuring apparatus. A successful prediction from quantum cosmology, for example concerning the distribution or constitution of the galaxies (Hartle and Hawking, 1983; Hawking, 1984), will put severe strain on the dogmatic elements of Bohr's interpretation. This is the second reason that we might suspect that the problems of quantum gravity and the interpretation of quantum mechanics to have something to do with each other.

To put the point slightly differently, there are two ways in which the problems of the interpretation of quantum mechanics might conceivably be resolved. The first is by progress in philosophy, so that one of the interpretations is shown, by some sort of philosophical argument, to be clearly superior to the others (for example by showing that it is the only one which is sensible). The other is by progress in physics, by which I mean the discovery that certain phenomena cannot be understood without modifying the quantum theory.

Clearly this first possibility is our only option if it turns out that there are no phenomena for which the quantum theory does not provide a complete description. If this is indeed the case then we are left in the situation of having to choose between the various radical modifications in our naive ontological or epistemological views which have been offered by the proponents of dogmatic interpretations of quantum mechanics. If none of these are appealing, our only hope lies in the second option, which is that developments in physics will require a modification in physical theory which will release us from the obligation to give a final interpretation to quantum theory.

It is the purpose of this paper to argue that the second possibility may be in the process of being realized, and that the phenomena whose explanation will require a modification in the quantum theory is in fact gravitation. I will argue that several examples which have been studied recently of systems involving both gravitational and quantum effects point to the breakdown of a distinction which is crucial for the dogmatic interpretations of quantum mechanics. This is the distinction between quantum fluctuations and ordinary statistical fluctuations. I will argue that what these examples point to is that this distinction can only be made unambiguously in the absence of a gravitational field, and in that case, only from the point of view of certain preferred observers. If this is the case then to bring together gravitation and quantum theory we need a generalized notion of a statistical state which can encompass both the pure quantum state and the ordinary statistical ensemble. I will then argue that the only simple way to do this is to accept the statistical interpretation of the wave function, so that the quantum state corresponds to a real ensemble of physical systems, just as does the ordinary thermal ensemble, and such that the quantum fluctuations are ordinary statistical fluctuations.

Of course, given a breakdown in the distinction between the quantum and the thermal ensemble, this may not be the only possibility. (It is, however, the only sensible one I know of at present.) But what is certain is that if this is the case it will not be possible to maintain that the quantum state gives a complete description of an individual system. That is, unless one is willing to make a similar assertion for the thermal ensemble; however, such an assertion would be demonstrably false. But if this can no longer be maintained then the dogmatic interpretations of quantum mechanics loose their appeal. For, as physicists, we are not likely to be satisfied with a fundamental theory that we know does not give a complete description of an individual system.

In the following sections I will discuss three arguments based on examples involving gravity which lead to the conclusion that quantum fluctuations must be considered to be ordinary statistical fluctuations. These are (1) the mixing of quantum and thermal effects under general coordinate transformations, (2) the evolution from pure to mixed states in black hole evaporation, and (3) the impossibility of distinguishing experimentally between quantum and statistical fluctuations of gravitational radiation.

However, before turning to these arguments I need to consider two points to avoid some possible misreadings of my intentions in these arguments. First of all, the examples that I will give are based on calculations which have been done in quantum field theory. The reader might then ask, how is it that calculations done within the quantum theory can be used as the basis of an argument asserting limits to the validity of the quantum theory? Is there not a risk of a contradiction here?

There is not, and to understand why one must appreciate an important fact, which is that quantum field theory, as presently formulated, is not generally covariant. While certain features of the theory, such as the Lagrangians which are used to specify the dynamics, are generally covariant, the definition of the vacuum state, and the related demarcation of particle and antiparticle states, depends on a preferred choice of a time coordinate. (The reason for this is discussed in the next section.) It turns out that this does not matter for special relativity, because these definitions are invariant under transformations between inertial observers in Minkowski space-time. But they are not invariant under any wider group of transformations in Minkowski space-time; further, in a general space-time there are no transformations under which these definitions are invariant.

How then can we do any computations involving both quantum effects and general relativity? The reason is that in Minkowski space-time we use a particular preferred set of time coordinates—those belonging to inertial observers. As we will discuss later, that we do this is an assumption, one which is made to get agreement with experiment. It does not follow from any other presently known physical principles. However, this lets us compute physical quantities both in Minkowski space-time, and in space-times which have an asymptotic region which is asymptotically flat.

The point is that the ability to carry out calculations in these special cases may give us the ability to describe the behavior of some special systems, but it does not give us what we require for a complete unification of general relativity with quantum mechanics. This would be a completely generally covariant formulation of quantum field theory, one that does not refer to any preferred classes of observers and which thus allows us to compute unambiguously the behavior of quantum systems in the absence of preferred time coordinates.

The question we are concerned with in this paper is how quantum mechanics may be extended in a way which satisfies both of these requirements. In examining this question we may use what we have learned from calculations making use of a preferred time coordinate, as long as we have some reason to believe that the result is physically correct. This will be the case if there is in fact some physical reason, such as a symmetry of the space-time geometry, which makes the preferred time coordinate special, so that we may expect that a generally covariant formalism would give the same results.

The second point is that I want to avoid the misimpression that I am claiming that there are no important distinctions between quantum fluctuations and ordinary statistical fluctuations. That there are is evidenced by the striking differences which are seen in the behavior of quantum and thermal systems. Here again we have two choices. If we assert that the quantum state gives the complete description of an individual system then the distinction is being made dogmatically, and as a result the differences between quantum and thermal systems are explained only by certain features of the quantum mechanical formalism, for example by the linearity of the Hilbert space representation. In this sense the differences are not explained.

On the other hand, if one regards the quantum state as a physical state describing an ensemble of systems then the striking features of the behavior of quantum fluctuations require explanation in physical terms. We want to ask, what physical processes could possibly give rise to fluctuations which display these peculiar properties?

That question is explored in two papers, the contents of which are closely related to this one. In one of them, the features which distinguish quantum fluctuations from more familiar statistical fluctuations are formulated in a way which suggests that gravitation may be intimately involved in whatever subquantum physics it is which is responsible for giving rise to the peculiar features of quantum phenomena (Smolin, 1982; the present paper is a revision of Chapter 3 of the original version of this reference). In the second, the question is explored through an examination of the postulates which go into Nelson's derivation of the Schrödinger equation from the theory of Brownian motion (Smolin, 1985; this is a revision of Chapter 3 of the original version of this reference).

2. THE MIXING OF QUANTUM AND THERMAL FLUCTUATIONS UNDER GENERAL COORDINATE TRANSFORMATIONS

Even in the absence of gravitational fields there is no generally coordinate invariant distinction between the quantum and thermal fluctuations of field quantities (Unruh, 1976; DeWitt, 1975; Fulling, 1973; Sanchez, 1979, 1981, 19 ; Sciama, 19 ; Sciama et al., 1981; Ashtekar and Geroch, 1974; Ashtekar and Magnon, 1975). This is perhaps best illustrated in terms of the well-known example of an accelerating detector in the presence of the ordinary Poincaré invariant vacuum state $|M\rangle$ of a quantum field in Minkowski space (Unruh, 1976). One can consider this physical situation from the point of view of any number of coordinate systems and the result will always be the same: the detector is excited to a temperature of

$$T_g = hg/2\pi c$$

The explanation for this effect, however, differs with the choice of coordinates.

If we choose to decompose the field with respect to annihilation and creation operators based on an inertial time coordinate the Minkowski vacuum $|M\rangle$ is found to be a state with no particles present. However, as was shown in a beautiful paper by Sciama (Sciama, 19), the excitation of the detector, from this point of view, can be understood in terms of the interactions of the quantum fluctuations of the dynamical variables for the detector with the ground state quantum fluctuations of the field variables. When the detector is moving inertially the decomposition of the quantum fluctuations into positive and negative frequency components with respect to the proper time of the detector agrees with the decomposition in terms of which the state of the field is being described. Thus, although the quantum fluctuations of the detector interact with the quantum fluctuations of the field, no net energy is exchanged between the detector and the field. This is because the ground state fluctuations consist equally of positive and negative frequency modes and both systems agree as to the distinction between them. However, when the detector moves noninertially the two definitions of positive and negative frequency no longer agree and the result is that the detector is excited. The important point is that this explanation for the excitation of the detector relies entirely on the interaction of the detector with the virtual quantum fluctuations in the vacuum state of the field.

But we can also describe the same physical situation in terms of a set of coordinates which are comoving with the accelerated detector. The simplest of these is the Rindler coordinate system, which is defined in terms of a set of uniformly accelerating clocks, whose motion is synchronized by demanding that they all share the same definition of simultaneity as our accelerating detector. From the point of view of a mode decomposition of the field made in terms of the Rindler coordinates, the Minkowski vacuum state $|M\rangle$ consists of a thermal distribution of quanta in equilibrium with a gravitational field with a temperature proportional to the local acceleration. Thus, in terms of these coordinates, the accelerating detector is at rest in a heat bath at temperature T_g and, as a result of its interactions with the quanta in the heat bath, it comes to equilibrium excited to a thermal spectrum at this temperature. Note that from the point of view of the Rindler coordinate system the excitation of the detector is described purely in terms of its interaction with the quanta of the heat bath, that is, entirely in terms of the effects of thermal fluctuations. Quantum fluctuations play no role.

Of course, these are only the two extremes in a multitude of possibilities. One might describe the situation from the point of view of a different set of Rindler coordinates, with respect to which the accelerated detector is not at rest. In terms of these coordinates the detector is now accelerating through a thermal bath in a static gravitational field, and the excitation of the detector is attributed to a combination of quantum and thermal effects. Another class of examples that one might pick are coordinate systems that agree with the Rindler coordinates in the region of interest, but differ from them in the region of the Rindler horizon, in such a way that these new coordinates may be extended in a nonsingular fashion to cover all of Minkowski space-time. Very general classes of such coordinate systems have been studied by Sanchez (1979, 1981, 19). One finds that in the region of the accelerated detector the Minkowski vacuum has exactly the same interpretation as in Rindler coordinates, it is a bath of particles in local thermodynamic equilibrium at temperature T_g in a static gravitational field, and the detector is excited as in the Rindler case. This makes it clear that the existence of the horizon and the incompleteness of the Rindler coordinates have nothing to do with the phenomena of the excitation of the detector.

More generally, given a decomposition of the field modes in terms of any coordinate system on Minkowski space-time the excitation of the detector will be ascribed in a completely coordinate-dependent fashion to some combination of quantum and thermal effects. But the physical result will always be the same.

Now in a situation like this, in which the phenomenon is coordinate independent but the explanation is coordinate dependent, we would like to try to understand the phenomenon in terms of coordinate-independent concepts, for any distinction between two effects that is dependent only on the choice of coordinates must not have any fundamental significance. If two kinds of phenomena are found to transform into each other under coordinate transformation then the two phenomena must be only two aspects of a single phenomenon, as in the case of electric and magnetic fields (Einstein, 1905/1916). As such, there should exist a coordinate-independent description of the phenomenon in question. Furthermore, if we want to describe quantum phenomena in the presence of arbitrary gravitational fields, there are no preferred coordinate systems, and we must learn to both speak about the phenomenon in question and to compute in a coordinate invariant manner.

If we are then to formulate quantum field theory in a coordinate invariant manner, then to begin with there will have to be some coordinate invariant extension of the concept of a state function. The behavior of fields in these invariantly specified state functions will exhibit fluctuations, but whether these fluctuations are virtual quantum fluctuations or statistically thermal fluctuations will not be a question which has physical meaning without the imposition from outside the system in question of a particular (and arbitrary) choice of coordinates. Thus, any absolute distinction drawn between quantum and thermal effects cannot refer to any actual distinction in nature, but will only be a property of a particular description based on some choice of coordinates.

Thus, in order to do quantum mechanics in the presence of arbitrary gravitational fields we shall need to invent a coordinate-independent concept that transcends the distinction between virtual quantum fluctuations of a quantum state and statistical fluctuations of a thermal state. How are we to do this?

Given the standard interpretation of quantum mechanics (Bohr, 1934; von Neumann, 1955; for a review of the problem of the interpretation of quantum mechanics, including references, see Jammer, 1974), in which the quantum state is supposed to be the complete description of an individual system, it is very difficult to see how this could be possible. For how could a virtual fluctuation in the description of an individual system be transformed, by a mere relabeling of coordinates, into a statistical fluctuation in a thermodynamic ensemble of physical systems?

This conceptual barrier is overcome if one adopts instead the view that the quantum state function, like the thermodynamic ensemble, corresponds to an ensemble of similarly prepared systems so that the quantum fluctuations are, like thermal fluctuations, real statistical fluctuations in the physical state of the system (Ballentine, 1970; Einstein, 1949; Jammer, 1974; Einstein et al., 1935; Bohm, 1952). For if quantum and thermal fluctuations are both real fluctuations in the physical state of the system then the distinction between thermal and quantum fluctuations is not a distinction between different kinds of objects, but only a question of which fluctuations, from the point of view of a specific observer, have a dissipative effect, and which do not. In the absence of a gravitational field and in the absence of a real thermal bath of particles one can choose a state of motion for the observer, from the point of which all fluctuations are nondissipative. These are then called quantum fluctuations, the definition of which might be fluctuations that in spite of their random character do not disorder. From the point of view of a different observer these same fluctuations act dissipatively, and thus are classified as thermal fluctuations. In the absence of a gravitational field but in the presence of a real thermal bath of particles an accelerated observer may see all of the fluctuations to be dissipative and hence classify them as thermal. But an inertial observer will see some of them as dissipative and some of them as nondissipative and will hence say that he or she is in the presence of both a vacuum state and a real bath of particles. Indeed this could be an operational definition of the distinction between the vacuum state and real particle states in the absence of gravitational fields. If one can choose an observer such that all of the fluctuations are seen to act nondissipatively then one is in the presence of the vacuum state. If there is no state of motion for the observer which eliminates the dissipative effects of the fluctuations, then those fluctuations (in the absence of gravitational fields) correspond to a thermal bath of real particles.

It is very interesting to wonder if there is from this point of view the possibility of a solution to the problem of the cosmological constant, which may be thought of as the question of why the gravitational field does not couple to vacuum fluctuations. Perhaps when we understand the relationship between the gravitational field and quantum fluctuations it will turn out that the gravitational field can only respond to fluctuations whose dissipative effect cannot be transformed away locally. To see that this is not so crazy consider that from this point of view what we have to understand the relationship between quantum and gravitational phenomena is why there is a source of universal noise in the evolution of local dynamical variables whose dissipative effects can be transformed away locally by the choice of a preferred class of motions, and, further, why this preferred class of motions is always the same preferred class in which the local effects of the gravitational field can be transformed away. It is perhaps not surprising that if the defining character of quantum fluctuations is that certain of their effects can be transformed away by going to the inertial frames, the quantum fluctuations do not couple to the gravitational field, whose defining characteristic is that certain of its effects can be transformed away by going to those same inertial frames.

Indeed, there is a very close analogy between making the distinction between quantum and thermal effects dependent on whether their effects can or cannot be transformed away by going to a preferred class of observers and the fact that the distinction between gravitational and inertial effects, which in terms of an arbitrary observer has no definite meaning, is made in terms of the fact that locally the gravitational effects can be transformed away by going to a preferred class of observers, while the inertial effects cannot. Indeed, if we now consider the case where gravitational fields are present it is clear that only for quantum fluctuations which occur on a scale that is small compared to the local radius of curvature will it be possible to transform away their dissipative effects. In other words in the presence of a real gravitational field it is precisely for those phenomena that occur on a scale such that the effect of the gravitational field cannot be transformed away that it will be impossible, by any choice of coordinates, to make a distinction between quantum and thermal effects.

Here, then are a number of reasons to take the view that quantum fluctuations are statistical fluctuations. First, it makes it possible to overcome the conceptual barriers to the construction of a generally covariant quantum field theory. Second, one has a new point of view towards the problem of the relationship between quantum and gravitational phenomena that was not available to us before. From this new point of view we can see new ways in which old and difficult problems, like the problem of the cosmological constant, might be solved. Third, this new point of view opens up new questions that may lie at the heart of the relationship between quantum and gravitational phenomena that we would not have asked before. For if this point of view is accepted the most interesting question concerning the relationship between quantum and gravitational phenomena is then: What is the connection between quantum and gravitational phenomena such that the relevant choices of preferred frames in each of the cases are necessarily the same?

Clearly quantum mechanics cannot answer this question because in quantum mechanics the relationship between the definition of the vacuum and the principle of inertia is put in by hand. There are infinitude of different assumptions one might make, for example, any of those considered by Sanchez (1979, 1981, 19). Thus, here is still another reason to prefer this point of view: because, once it is accepted, the problem of quantum gravity is not a technical problem of fitting two old theories together but is a problem in which new questions challenge us to invent new ideas with which to construct a new theory.

3. THE EVOLUTION FROM PURE TO MIXED STATES IN BLACK HOLE EVAPORATION

The observation that unitary deterministic evolution of pure quantum states might break down during the course of black hole evaporation (Hawking, 1974, 1975) was first made by Steven Hawking in 1976 (Hawking, 1976). Since that time a great many arguments have been made, both for and against this conclusion. While Hawking's argument is rather compelling it cannot be considered definitive, as it depends on certain assumptions about processes which take place at Planck scales, during the last stages of black hole evaporation. In the absence of a good quantum theory of gravity we cannot be sure of what really does go on at these scales, thus the argument concerning the loss of information during black hole evaporation is unlikely to be definitively resolved soon.

I will assume that the reader is familiar with the original discussions concerning this problem, by Hawking (Hawking, 1976) and others. This section will then be devoted to a consideration of the implications for the foundations of quantum mechanics, given the assumption that Hawking's original argument is correct. However, before turning to this discussion I will mention two points in favor of Hawking's conclusion, which are not, to my knowledge, in the literature.[2]

---

[2] Perhaps the best argument for loss of information in the literature is contained in a paper by Wald (1984).

3.1. Comments Regarding the Conjecture of Loss of Coherence in Black Hole Evaporation

While some scenarios have been suggested in which information is not lost in black hole evaporation, what has never been done is to show that these scenarios are plausible, given some reasonable assumptions about the effects of quantum gravitational effects on the evolution of black holes. While this is not the place for a full discussion of this matter, it is perhaps appropriate to indicate here why this may be a difficult thing to accomplish.

We may begin with the observation that it seems very likely that if a quantum theory of gravity is to avoid the loss of information in black hole evaporation, then, in that theory, quantum effects must eliminate the singularity in the center of the black hole, replacing it with some stable structure which can store an arbitrary amount of information, using an arbitrarily small amount of energy, as measured asymptotically outside the black hole. While it is not known whether there exists some quantum theory of gravity which can accomplish this, it is known that semiclassical quantum effects are not strong enough to eliminate singularities (Fischetti et al., 1979; Hartle and Hu, 1979, 1980a, 1980b). These semiclassical effects are dominated, in the limit of short distances, by dimension-four operators, as is, in fact, to be expected of the short distance behavior of any quantum field theory defined through a process of renormalization in four dimensions. Thus, it may be argued that if there is a quantum theory of gravity which eliminates singularities its high-energy behavior must be very different from that given by the dimension-four operators. Thus, we learn that if a quantum theory of gravity is going to eliminate singularities and thus avoid the loss of information in black hole evaporation, it is very likely to have a short distance behavior which is very different from that of a conventional quantum field theory.[3]

A second difficulty which must be faced is that in any scenario which saves the quantum state in black hole evaporation the relevance of the Hawking entropy for the thermodynamic and statistical mechanical properties of a black hole becomes doubtful. Since the entropy of an object is, by

---

[3] One is then tempted to consider whether a string formulation of quantum gravity might resolve the problem of the loss of information. This problem has been considered in Bowick et al. (1985), using some interesting facts which have been discovered concerning the statistical mechanics of the higher string excitations Bowick and Wijewardhana (1985). The conclusion is that the higher string states may play a role in the last stages of black hole evaporation, but only if loss of information has occurred so that the Hawking entropy is in fact the true entropy which governs the statistical mechanics of the situation when the black hole approaches the Planck scale. If loss of information has not occurred the excited modes of the string will have vanishingly small entropy compared to the true entropy of the black hole remnant, and will not play an important role.

statistical mechanics, supposed to be the logarithm of the number of quantum states accessible to the system, given fixed values of the macroscopic parameters, it is also a measure of the amount of information that could be stored in the object. However, the Hawking entropy, being related to the area of the black hole, decreases as the black hole evaporates. Thus, as measured by the Hawking entropy, the amount of information which could be contained in a black hole decreases as the black hole evaporates.

Let us consider, for example, a black hole, originally of mass $M \gg m_{\text{Planck}}$, which has, by some time, t, evaporated to a much smaller mass m which is perhaps 1000 Planck masses. The remaining mass of the black hole has gone into thermal radiation, which has, at this time, an entropy of the order of $4\pi(M-m)^2$. Now, in order to restore a pure state, we need an amount of information equal to this value. This information is known to be contained in the phase correlations which exist between the photons which escape to infinity to make up the Hawking radiation and the photons which fell into the black hole. Since these photons are paired, so that the correlations exist between the members of each pair, if the pure state is to be restored by the specification of this phase information, it must be that there is one photon with an undetermined phase inside the black hole for each one in the Hawking radiation. Thus, if it is possible to restore the pure state the entropy of the black hole must contain a term which counts this undetermined phase information, which will be of the order of $4\pi(M-m)^2$. However, by the time t the Hawking entropy of the black hole is much smaller than this.

Another way to put this is that the black hole must contain an amount of information in its external structure which is equal to its true entropy. In order to store the amount of information which is required to specify the missing phase information of the photons that fell in the black hole at time t must have at least $\exp[4\pi(M-m)^2]$ different possible internal states, which are compatible with its value of mass, charge, and angular momentum. But, if the Hawking entropy is a real statistical entropy the number of such states is only $\exp[4\pi m^2]$, which is enormously fewer.

Thus, if information is not lost during black hole evaporation, the Hawking entropy is not the true entropy of a black hole. If this is the case the true entropy of a black hole is a quantity which is not determined by the properties of a black hole which can be measured from outside the hole. Instead, it depends on the details of the history of the black hole. For example, two Schwarzchild black holes of a given mass m, one of which was just formed at that mass, the other of which was originally much larger, but reached this mass through an evaporation process, would have equal Hawking entropies, but very different true entropies. If we put one of these black holes in a box with radiation, the Hawking entropy would be relevant only for thermodynamic processes on scales short compared to the evaporation time, $t_e = t_{\text{Planck}} m^3/m^3_{\text{Planck}}$. The nature of the thermodynamic equilibrium which would be reached after an arbitrarily long time would be governed by the true entropy and not the Hawking entropy.

It must be stressed that there is nothing in this possibility which is inconsistent with what we know presently. However, what we learn from these considerations is that one of the prices of insisting that information is not lost in black hole evaporation must be that the true thermodynamic properties of a black hole, given to us at some unknown time during its history, cannot be determined by any observations made outside of its horizon.

3.2. Consequences of the Loss of Information During Black Hole Evaporation

In the following, we will assume that information really is lost during black hole evaporation, and turn to a discussion of the consequences of this for physics.

At first sight it seems that the loss of phase information during black hole evaporation need not entail any radical modification of quantum theory. For example we may follow the suggestion of Hawking and replace the notion of the deterministic evolution of the pure state by a notion of deterministic evolution of density matrices (Hawking, 1976). On a formal level it is perfectly possible to proceed in this direction, but let us examine a little more closely just what this means for our understanding of physics.

First of all, before we become too entangled in formal developments, we should remind ourselves that an individual physical system is never in a mixed state. In this respect "state" is rather a misnomer, for what a mixed state corresponds to is not anything in the physical world but, rather an ensemble of distinct physical states in which the physical system in question might be found. In making up the mixed state these states are weighed by probabilities which reflect nothing in the real world but rather indicate our state of knowledge and ignorance about the world.

The reader may think that the above belabors the obvious; however, the point is that by accepting a proposal for a dynamics of mixed states we are renouncing, in a way that quantum mechanics does not, the two claims that physics can give a complete specification of the state of an individual system and that physics can give us a deterministic evolution for the states of individual systems. Quantum mechanics, although it gives only a probabilistic connection between the state of a system and the results of interactions between the system and macroscopic measuring apparatuses, fulfills these claims no less rigorously than Newtonian mechanics. Thus, we should not mistake Hawking's proposal for a mere formal modification of the quantum theory; it calls for a renunciation of the meaning of physics far more radical than that called for by the usual (Copenhagen) interpretation of quantum mechanics (Bohr, 1934; von Neumann, 1955).

To put it differently, no physical system can evolve from a pure state into a mixed state. Physical systems evolve from one state to another, which are all, by definition, pure. What we are faced with is that if quantum mechanics can be modified so as to incorporate gravitational phenomena it will necessarily lose the property of being able to predict which quantum state a given individual state will evolve to.

The question is then, how are we to construct a theory which can incorporate the loss of quantum phase information in black hole evaporation without giving up the possibility of giving a complete description of the state of an individual physical system?

In order to look at this more closely, let us examine in more detail the process by which a pure state evolves to a mixed state during the course of the evolution of an evaporating black hole. The Hawking radiation arises initially as a quantum fluctuation of the field in which a particle-antiparticle pair is created. One member of the pair may fall inside the horizon, in which case it acquires negative energy with respect to observers at infinity. This allows the second member of the pair to escape to infinity as a real quanta carrying some positive amount of energy. Because we can only observe those particles that remain outside the horizon a density matrix constructed to describe the state of those particles must be a mixed state. Further, it is also true that because of the special properties of the geometry near the space-time horizon, the state is actually a thermal state.

Of course, there is nothing unusual about the necessity of describing an incomplete piece of a quantum system by a mixed state, as the pure state which describes the entire system can normally be reconstructed by including the information about the part of the system that is not being observed including the quantum correlations which may exist between the two systems. But what may happen in the black hole case is that the other part of the system ceases to exist altogether, in which case information really has been lost and the mixed state is the most complete description that can be given concerning the state of the system after the black hole has evaporated away.

Now, because the original particle antiparticle pair arises as a quantum fluctuation in the vacuum state of the field in the region of the horizon, we may describe this situation by saying that quantum fluctuations in one region of space-time have, as a result of the causal structure of the black hole space-time, evolved to give rise to random statistical fluctuations in another region of space-time. The randomness of the distribution of energy in the thermal radiation at late times is a direct consequence of the randomness in the distribution of quantum fluctuations in the vacuum. Indeed, we might say that the unusual causal structure of the black hole is serving as a kind of microscope for looking at the structure of the quantum field theory vacuum in the neighborhood of the horizon. It is then perhaps not surprising that this view of the vacuum fluctuations seen with the aid of the black hole is exactly the view of the vacuum in the absence of gravitational fields seen by an observer with an acceleration equal to the surface gravity of the black hole. Perhaps, given the hypothesis above that the distinction between real energy and vacuum energy is whether or not the dissipative effects of the fluctuations can be transformed away by a choice of coordinates, one way to think about what has happened is that the black hole has acted as a source of energy to the fluctuations so that they can propagate out to infinity where they are seen as real particles by an inertial observer.

But, this is getting a little ahead of the discussion. The question is, how are we to make sense of a situation in which the quantum fluctuations at one time give rise to thermal fluctuations at a later time? Similarly, how can we understand how a pure state can evolve into a mixed state? If we accept the notion that the quantum state gives the complete description of an individual system it seems that we must conclude that this will not be possible. For, on the standard interpretation quantum and thermal fluctuations are completely different kinds of objects. Quantum fluctuations are not real fluctuations in nature, but only terms in a perturbative description of a single, well-defined state. If we consider the evolution of a quantum system in terms of its exact Hamiltonian, then quantum fluctuations as such never enter the picture. There is, of course, the inevitable dispersion in the results of measurements of noncommuting operators, but in this case the randomness is usually understood to be induced by the interaction with the measuring instrument. In the absence of measurement the state evolves deterministically according to the Schrödinger equation. On the other hand thermal fluctuations are real statistical fluctuations in the physical state of the system due to the degradation of some component of the system's energy into random motion. It is then very difficult to understand how, given this conventional view, the distinction between quantum and thermal fluctuations could be other than absolute, and then it is a little mysterious that one could evolve into the other.

Similarly, on the usual interpretation of quantum mechanics, a pure state gives the complete description of the state of an individual system. A thermal state, on the other hand, is a statistical description of an ensemble of individual systems. It seems then very unlikely that one could have a well-defined sense in which pure states evolve to mixed states—they are not even descriptions of the same kinds of systems. That is, while we can say that the unitary evolution of the quantum state has broken down and become indeterminate, we cannot say that a pure state has evolved into a mixed state, for the initial pure state described an individual system, and, no matter what else may be wrong, an individual system cannot involve into the ensemble of systems that is needed to give meaning to the mixed state.

However, if we accept the notion that the quantum state itself is a statistical description of the state of an individual system, then these difficulties do not occur. Quantum fluctuations can result in statistical fluctuations at a later time because quantum fluctuations are already real statistical fluctuations. What has happened during the evolution of the system is only that one source of statistical uncertainty, that associated with the quantum state description, has become converted into uncertainty in the choice of quantum state to represent the system after the evaporation of the black hole. If the quantum state description is understood as referring itself to a statistical description of an ensemble of physical systems, then it is not too surprising if under certain situations uncertainty as to which physical state the system is in (among an ensemble represented by a given quantum state) can lead to uncertainty as to which quantum state the system may be in. As far as the actual state of an individual system is concerned there is no difficulty. It simply evolves, according to laws we do not know. What has happened is that the procedure of considering the system to be a member of a particular ensemble of similarly prepared systems—what we call a pure state—is in this situation not as useful for predicting the results of certain kinds of experiments as it is in other situations.

Furthermore, if we accept this notion that quantum states give only a statistical description of the state of an individual system there is no reason to believe that the loss of information during black hole evaporation has any implication for the possibility of constructing, sometime in the future, a complete description of the evolution of individual physical systems. Of course we may not be interested in constructing such a description. But if we are then the existence of a phenomenon in which the deterministic evolution of pure quantum states breaks down implies that this description would have to be a more detailed description of the state of the system than that given by quantum mechanics.

4. THE IMPOSSIBILITY OF DISTINGUISHING EXPERIMENTALLY BETWEEN PURE AND MIXED STATES OF THE GRAVITATIONAL FIELD

So far we have seen that it is not possible to make a coordinate invariant distinction between the effects of pure quantum and ordinary statistical fluctuations. We have also seen that it is very likely that in some circumstances involving strong gravitational fields, quantum fluctuations at one time can lead to random thermal fluctuations at a later time. Now I would like to mention a third way in which the distinction between quantum fluctuations and ordinary statistical fluctuations is weakened when gravitation is relevant.

The application of quantum mechanics to a given physical system is based on the assumption that it is possible by measuring a complete set of commuting observables, to determine experimentally which pure state a system is in. Now, it is actually possible, in some situations, to measure more information concerning a system than is necessary to determine a given pure state. However, more information than is needed to determine a pure state cannot, because of the existence of the quantum fluctuations, be relevant to any predictions made about the future evolution of the system. That is, if one tries to measure more than a complete set of commuting observables one will find that no matter what state a system is prepared in, there will always be an irreducible random dispersion in the results of the experiments. Thus, for normal quantum systems, the boundary between the pure and the mixed states coincides with the limit of how accurately the evolution of the observables of the system can be predicted based on measurements which it is physically possible to make at a given time.

However, the same is not true for the case of the gravitational field, at least at the linearized level. Instead, it can be shown that it is impossible, in principle, to construct a measuring apparatus which could determine exactly which pure state the linearized gravitational field is in (Smolin, 1984, 1985). As the details of the demonstration of this statement are given in another paper (Smolin, 1985) I will only mention here the essential points of the argument.

The strength of the coupling of a body with the gravitational field is proportional to the internal stresses and pressure in the medium. These are bounded, by the positive energy condition, by the energy density of the medium. (Even if one could imagine quantum systems that violate the positive energy condition, it is clear that no stable material could do so, and detectors must be constructed out of stable material.) Thus, one can increase the strength of the coupling of a material to gravitational radiation by increasing the stiffness of the material until this limit is reached (at which point the speed of sound in the material is equal to the speed of light *in vacua.*) After this point is reached the coupling can only be increased by increasing the density of the material. However, one cannot do this forever, and it in fact turns out that the body will always undergo gravitational collapse before the efficiency of the detector to respond to gravitational radiation approaches one. Specifically, if L is the linear dimension of the body in the direction in which the gravitational wave is propagating, and $L_{\mathrm{abs}}(\lambda)$ is the absorption length for gravitational radiation at wavelength λ, one always finds that (Smolin, 1985)

$$\frac{L}{L_{\mathrm{abs}}(\lambda)}=\left(\frac{R_{\mathrm{schu}}}{L}\right)f(\lambda)<1 \tag{1}$$

where $f(\lambda)$ is a dimensionless function which differs for different processes, but which, assuming the positive energy condition, is always less than 1.

Thus, most of the gravitons in a given gravitational wave will pass through any given detector without exciting it. This means that even if one measures a complete set of commuting observables, for example the number operators N_k for each mode, one will not be able to determine very precisely after the measurement which state the gravitational field is in. Furthermore, as it is impossible to predict which gravitons will be the ones to excite the detector, the same pure state will, under repeated measurements, give different readings. The postulate of the repeatability of measurements of commuting operators breaks down.

In particular, it can be shown that it is, in general, impossible to distinguish experimentally a pure state with a given spectral distribution $\langle N_k \rangle$ with fixed phase relations from a chaotic state with the same $\langle N_k \rangle$ but with random undetermined phase relations (Smolin, 1985). This means that under repeated measurements of any particular observable it is impossible to distinguish between dispersion in the results caused by quantum fluctuations in the pure state from statistical fluctuations caused by the impossibility of determining which state the system is in. Thus, for the case of gravitational radiation there can be no absolute operational distinction between quantum fluctuations and statistical fluctuations.

Before finishing this discussion, two caveates must be introduced. The calculations on which the forgoing discussion is based were done in the context of linearized general relativity. This is certainly alright for the description of gravitational radiation; however, no account was taken of the possibility that the detector might utilize some effects involving strong gravitational fields. In particular, it can be shown that black holes, or objects very close to their critical radius for gravitational collapse, interact, in general, more strongly with gravitational radiation than ordinary matter. It seems unlikely that a black hole could be used as a detector, because all that it can emit is thermal radiation. However, one might imagine that an object extremely close to its gravitational radius might do.

While this point has not been settled, it is important to point out that, in general, such objects are beset, in the presence of external perturbations, by instabilities, which will make it impossible to achieve an efficient detector in this manner. For example, Garfinkle and Wald (1985) have shown that it is possible using a charged shell with e slightly greater than M to construct a box to contain gravitational radiation. They were also able to show that the shell was stable under small perturbations in its radius. However, Dell (1985) has shown that, because of the enormous blueshift suffered by external radiation as it falls to the surface of the shell, their "box" is unstable in the presence of external radiation, and is thus not suitable for use as a detector of external gravitational radiation. Of course, at the present time we cannot rule out the possibility that some other clever way will be found in which a detector incorporating strong gravitational fields in its construction could be used as an efficient detector for gravitational radiation. We may hope that this question will be resolved, one way or the other, in the near future.

Second, the argument given in this section in no way implies that quantum mechanics is inapplicable to the gravitational field. It is at best an argument that the Hilbert space formulation cannot give a complete description of quantum gravitational effects. Even as such, one cannot go from a statement that a particular concept or quantity in a physical theory cannot be operationalized to the statement that it does not exist in reality without additional assumptions. The argument, used by Heisenberg in his advocacy of quantum mechanics as a complete physical theory, that distinctions between physical systems that cannot be realized experimentally should not appear in theory, is based on a number of philosophical preconceptions that it is possible to disagree with. It is perfectly possible, in spite of the uncertainty principle, to believe that quantum mechanics is an incomplete theory and to believe specifically in the paths of individual electrons.

However, to the extent that the belief in the completeness of quantum mechanics is based on the argument that theory should not make distinctions that cannot be operationalized, it must be acknowledged that the same argument leads, in the present situation, to the conclusions that for the case of the linearized gravitational field physics should not make an absolute distinction between the effects of quantum and statistical uncertainties, or between pure states and mixed states.

5. CONCLUSIONS

In this paper I have given three examples, based on calculations which may be considered reliable, in which the distinction between what is a quantum effect and what is an ordinary statistical effect seems to be breaking down in situations in which relativistic gravitational effects are present. I have argued that what these examples are trying to tell us is that in order to construct a theory which correctly describes situations in which both gravitational and quantum effects are important we must accept that this distinction cannot be made absolutely in a way which is independent of the choice of a coordinate system. If we want physics to be coordinate independent (which seems a necessary requirement as space-time is no more likely to come equipped with coordinates drawn on it in quantum physics than in classical physics) then we must go to an interpretation of quantum mechanics in which quantum fluctuations are a variety of ordinary statistical fluctuations.

What are the implications, for physics, of these conclusions. Clearly what is at stake here is more than just which interpretation of quantum mechanics is preferable; if the result is to be more than just a verbal exercise these arguments must lead to new insights concerning the relationship between quantum and gravitational phenomena, and to new predictions of observable phenomena. The task then is twofold: (1) to investigate whether these ideas may lead to the formulation of a theory of quantum processes involving gravitation which is sensible and generally coordinate invariant, and (2) given the difficulty of this problem to try to progress by looking for effects which might indicate a breakdown of quantum theory in regimes where the difficulties discussed in this paper are relevant.

Some work in these directions has been attempted in the last several years. Attempts have been made to generalize the stochastic formulation of quantum theory (Nelson, 1966) to circumstances involving weak gravitational fields (Smolin, 1984c, 1985), and further work in this direction is in progress. One characteristic of the results which have been found is that nonlinear effects come into the evolution of the wave function exactly when relativistic gravitational effects, and in particular the ambiguities in the definition of time, become important.

A class of nonlocal hidden variable theories has also been constructed (Smolin, 1985, 1983). While these do not directly involve gravity, they have one interesting feature, which suggests that gravitational effects may be of the same order as effects which represent deviations from the standard quantum theory. The basic idea which leads to this conclusion is that, to be in accord with the experimental situation regarding Bell's theorem (Aspect et al., 1981, 1982), the hidden variable theory must be nonlocal. We may then set the theory up in such a way that the basic interactions are nonlocal, but a classical theory involving only local interactions emerges in a thermodynamic limit in which N, the number of particles, is taken to infinity. However, suppose that in our universe N is finite, although very large, of order 10^{80}. Then one would expect fluctuations around this classical limit, in which the nonlocal effects become relevant, and, by the usual arguments, the scale of these fluctuations should be of order $1/\sqrt{N}$, about 10^{40}. Thus if it only becomes possible to localize a quantum as a classical particle in the limit $N \rightarrow \infty$ for finite N, localization should be possible only to a scale which is $1/\sqrt{N}$ of the average distance between particles in the universe.

Now, the ratio of the electron Compton wavelength to the Hubble distance is approximately equal to $1/\sqrt{N}$ of the number of electrons within the Hubble distance. Thus, it is tempting to try to identify these $1/\sqrt{N}$ fluctuations with the quantum fluctuations.

Indeed, it is exactly this scheme which was found to work in a model of a nonlocal hidden variable theory for quantum mechanics (Smolin, 1985, 1983). What is perhaps most interesting, however, is that, as was alluded to above, we would expect effects which deviate from quantum mechanics to occur at the next order in an expansion in $1/\sqrt{N}$, but this is exactly the scale of the ratio of gravitational to electromagnetic forces for elementary particles.

Thus, the results which have been found so far suggest that corrections to quantum evolution, coming from whatever fundamental theory quantum mechanics is the statistical mechanics of, will become important at the same scale that quantum gravitational effects become nonnegligible. If this is really the case then the difficulties, such as the ultraviolet problems, which are encountered in attempts to directly quantize the gravitational field are probably less relevant to the ultimate solution to the problem of quantum gravity than are the issues of principle which we have been discussing here.

Finally, as a last remark, we may note that if, as we argued in Section 3, quantum coherence breaks down in black hole evaporation, and if, as Hawking has argued (Hawking, 1974, 1975), virtual black holes are an important component of the vacuum at Planck scales, the resulting loss of coherence could have the effect of weakening the contributions from virtual fluctuations of fields at shorter than the Planck scale, providing a natural regulator for quantum field theory (Crane and Smolin, 1985a, b). Thus, it may even be that the short-distance problems which arise when one tries to construct a renormalizable theory of quantum gravity may have their solution in a deeper understanding of the problems we have been discussing here.

ACKNOWLEDGMENTS

This work was begun during a stay at the Institute for Theoretical Physics at Santa Barbara and a first draft of this paper was written at the Institute for Advanced Study in Princeton. After this, my thinking on these matters was sharpened by many discussions with the members of the relativity group at the University of Chicago. In addition, I would like to thank many colleagues who made comments on the first draft of this paper including Julian Barbour, John Dell, Freeman Dyson, James Hartle, Ted Jacobson, Edward Nelson, and Andrew Strominger.

This research was supported in part by the NSF under grant Nos. PHY-77-27084, PHY-80-26043, and PHY-85-46627 and the DOE under Contract No. DE-AC02-76ER02220.

REFERENCES

Ashtekar, A., and Geroch, R. P. (1974). *Reports on Progress in Physics*, **37**, 1211.

Ashtekar, A., and Magnon, A. (1975). *Proceedings of the Royal Society of London Series*, **A346**, 375

Aspect, A., Grangier, P., and Roger, G. (1981). *Physical Review Letters*, **47**, 460;

Aspect, A., Grangier, P., and Roger, G. (1982a). *Physical Review Letters*, **49**, 91.

Aspect, A., Dailard, J., and Roger, G. (1982b). *Physical Review Letters*, **49**, 1804.

Ballentine, L. E. (1970). *Reviews of Modern Physics*, **42**, 358.

Bohm, D. (1952). *Physical Review*, **85**, 166, 180.

Bohr, N. (1934). *Atomic Theory and the Description of Nature.* Cambridge University Press, Cambridge.

Bowick, M., and Wijewardhana, R. (1985). *Physical Review Letters*, **54**, 2485.

Bowick, M., Smolin, L., and Wijewardhana, R. (1985). Yale University preprint in preparation.

Crane, L., and Smolin, L. (1985a). *General Relativity and Gravitation*, to appear.

Crane, L., and Smolin, L. (1985b). *Nuclear Physics* **B**, to appear.

Dell, J. C. (1985). University of Windsor preprint in preparation.

DeWitt, B. S. (1975). *Physics Report*, **19C**, 297.

DeWitt, B. S., and Graham, N., eds. (1973). *The Many Worlds Interpretation of Quantum Mechanics.* Princeton University Press, Princeton, New Jersey.

Einstein, A. (1905/1916), in *The Principles of Relativity: A Collection of Individual Memoirs*, Lorentz *et al.*, eds., Dover, New York.

Einstein, A. (1949). In *Albert Einstein: Philosopher-Scientist*, P. A. Schilpp, ed. Library of Living Philosophers, Evanston, Illinois.

Einstein, A., Podolsky, B., and Rosen, N. (1935). *Physical Review*, **47**, 777.

Everett, H., III (1957). *Reviews of Modern Physics*, **29**, 454.

Fischetti, M. V., Hartle, J. B., and Hu, B. L. (1979). *Physical Review*, **D20**, 1757.

Fulling, S. A. (1973). *Physical Review*, **D7**, 2850.

Garfinkle, D., and Wald, R. M. *General Relativity and Gravitation*, **17**.

Geroch, R. (1984). *Nous*, **18**, 617.

Hartle, J. B., and Hawking, S. W. (1983). *Physical Review*, **D28**, 2960.

Hartle, J. B., and Hu, B. L. (1979). *Physical Review*, **D20**, 1772.

Hartle, J. B., and Hu, B. L. (1980a). *Physical Review*, **D21**, 2756.

Hartle, J. B., and Hu, B. L. (1980b). *Physical Review*, **D22**, 2091.

Hawking, S. W. (1974). *Nature*, **248**, 30.

Hawking, S. W. (1975). *Communications in Mathematical Physics*, **43**, 199.

Hawking, S. W. (1976). *Physical Review*, **D14**, 2460.

Hawking, S. W. (1984). *Nuclear Physics*, **B239**, 257.

Jammer, M. (1974). *The Philosophy of Quantum Mechanics—The Interpretations of Quantum Mechanics in Historical Perspective*, John Wiley and Sons, New York.

Nelson, E. (1966). *Physical Review*, **150**, 1079.

Nelson, E. (1967). *Dynamical Theories of Brownian Motion.* Princeton University Press, Princeton, New Jersey [In *Lecture Notes in Physics*, No. 100 (Springer-Verlag, Berlin, 1979)].

Sanchez, N. (1979). *Physics Letters*, **87B**, 212.

Sanchez, N. (1981). *Physical Review*, **D24**, 2100.

Sanchez, N. (1986). Meudon reports (unpublished).

Sciama, D. W. (1979). Thermal and quantum fluctuations in special and general relativity: An Einstein synthesis. In *Centenario di Einstin* (1879–1979). Edditrice Giunti Barbera Universitaria.

Sciama, D. W., Candelas, P., and Deutch, D. (1981). *Advances in Physics*, **30**, 327.

Smolin, L. (1982). On the nature of quantum fluctuations and their relation to gravitation and the principle of inertia, IAS preprint, November 1982, revised July 1985.

Smolin, L. (1983). IAS preprint, July 1983.

Smolin, L. (1984a). In *Quantum Theory of Gravity* (*the DeWitt Festschrift*), S. Christensen, ed. Adam Hilger, Bristol, England.

Smolin, L. (1984c). Essay submitted to the Gravity Research Foundation, 1984, and EFI preprint (March 1984).

Smolin, L. (1985a). An equivalence principle for stochastic quantum mechanics, Yale preprint, July 1985.

Smolin, L. (1984b). *General Relativity and Gravitation*, **16**, 205.

Smolin, L. (1985c). *Contribution to the Oxford Quantum Gravity Discussion Conference, March 1984*, C. J. Isham and R. Penrose, eds. Oxford University Press, London, England.

Smolin, L. (1985b). *General Relativity and Gravitation*, **17**, 417.

Unruh, W. G. (1976). *Physical Review*, **D14**, 870.

von Neumann, J. (1955). *Mathematical Foundations of Quantum Mechanics.* Princeton University Press, Princeton, New Jersey.

Wald, R. M. (1984). In *Quantum Theory of Gravity*, Steven M. Christensen, ed. Adam Hilger Ltd., Bristol, England.